Cambridge Assessment Admissions Testing

STEP MATHEMATICS

SPECIFICATIONS for June 2019 Examinations

General Introduction

There are three specifications for STEP: one for Mathematics 1, one for Mathematics 2 and one for Mathematics 3.

The three specifications have been written to follow, in the ways set out below, the content of the Department for Education's A level Mathematics[1] and the Pure content of AS and A level Further Mathematics[2] specifications; however some topics have been removed and some additional topics have been included. In the case of Mathematics 2 and Mathematics 3, additional sections have been included outlining which Probability, Statistics and Mechanics topics might be tested. Whilst most questions will be set on areas mentioned in the respective specification, questions may also be set on areas that are not explicitly mentioned; when this is the case, appropriate guidance will be given in the question.

	Pure	**Mechanics**	**Probability/ Statistics**	**Notes**
Mathematics 1	Pure content of A level Mathematics with some modifications[3] and additions.	Mechanics content of A level Mathematics with some modifications[3] and additions. Assumed: Pure content of Mathematics 1.	Probability/Statistics content of A level Mathematics with some modifications[3] and additions. Assumed: Pure content of Mathematics 1.	Additions are indicated in the specification by ***bold italics***.
Mathematics 2	The prescribed Pure content of AS Further Mathematics with some modifications[3] and additions. Assumed: Pure content of Mathematics 1.	Additional topics as outlined. Assumed: Pure and Mechanics content of Mathematics 1; Pure content of Mathematics 2.	Additional topics as outlined. Assumed: Pure and Probability/Statistics content of Mathematics 1; Pure content of Mathematics 2.	Additions are indicated in the specification by ***bold italics***.
Mathematics 3	The prescribed Pure content of A level Further Mathematics with some modifications[3] and additions. Assumed: Pure content of Mathematics 1 and 2.	Additional topics as outlined. Assumed: Mechanics content of Mathematics 1 and 2; Pure content of Mathematics 1, 2, 3.	Additional topics as outlined. Assumed: Probability/Statistics content of Mathematics 1 and 2; Pure content of Mathematics 1, 2, 3.	Additions are indicated in the specification by ***bold italics***.

[1] https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics

[2] https://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics

[3] A few topics have been removed and, occasionally, wording from the DfE document has been modified for clarity.

Format of the papers

Mathematics 1 will be a 3-hour paper divided into two sections.

The paper will comprise 11 questions:

Section A (Pure Mathematics)	eight questions
Section B (Mechanics, and Probability/Statistics)	three questions, with at least one on Mechanics and at least one on Probability/Statistics

Mathematics 2 and Mathematics 3 will each be a 3-hour paper divided into three sections.

Each paper will comprise 12 questions:

Section A (Pure Mathematics)	eight questions
Section B (Mechanics)	two questions
Section C (Probability/Statistics)	two questions

Each question will have the same maximum mark of 20. In each paper, candidates will be assessed on the six questions best answered; no restriction will be placed on the number of questions that may be attempted from any section.

The marking scheme for each question will be designed to reward candidates who make good progress towards a complete solution. In some questions a method will be specified; otherwise, any correct and appropriately justified solution will receive full marks whatever the method used.

Candidates' solutions must be clear, logical and legible; marks may be lost if examiners are unable to follow a candidate's working.

Specifications

These specifications are for the guidance of both examiners and candidates. The following points should be noted:

1. Whilst most questions will be set on topics mentioned in the specifications, questions may also be set on areas that are not explicitly mentioned, or in ways that extend topics that are mentioned; when such questions are set, candidates will be given appropriate guidance in the question.

2. Individual questions will often require knowledge of several different specification topics.

3. Questions may test a candidate's ability to apply mathematical knowledge from the specifications in unfamiliar ways.

4. Questions may be set that require knowledge of topics from the higher tier GCSE Mathematics.[4]

5. Solutions will frequently require insight, ingenuity, persistence, and the ability to work through substantial sequences of algebraic manipulation.

6. Examiners will aim to set questions on a wide range of topics, but it is not guaranteed that every topic will be examined every year.

7. The Pure sections of each specification assume knowledge of the full Pure content of all preceding specifications.

8. The Mechanics and Probability/Statistics sections of each specification assume knowledge of the appropriate Pure Mathematics for that specification, and of the full Pure content of all preceding specifications. In addition, each Mechanics section assumes knowledge of the Mechanics sections of preceding specifications, and similarly for Probability/Statistics sections.

9. ***Bold italics*** are used to indicate additional topics that do not fall under the compulsory content set out in the relevant government document. For Mathematics 2 and Mathematics 3 this includes all additional topics in the Mechanics and Probability/Statistics sections.

Formulae booklets and calculators

Candidates will not be issued with a formulae book. Formulae that candidates are expected to know are listed in the appendix to this document. Other formulae will be given in individual questions, should they be required.

The required formulae for STEP extend beyond those required for the corresponding A levels.

Calculators are not permitted (or required).

Bilingual dictionaries may be used.

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[4] https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/254441/GCSE_mathematics_subject_content_and_assessment_objectives.pdf

MATHEMATICS 1

Section A: Pure Mathematics

Content

Proof

Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion, ***proof by induction***.

Understand and use the terms 'necessary and sufficient' and 'if and only if'.

Disproof by counter-example.

Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).

Algebra and functions

Know, understand and use the laws of indices for all rational exponents.

Use and manipulate surds, including rationalising the denominator.

Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown.

Solve simultaneous equations in two (***or more***) variables by elimination and by substitution; including, ***for example***, one linear and one quadratic equation.

Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions.

Express solutions through correct use of 'and' and 'or', or through set notation.

Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically.

Solve inequalities and interpret them graphically; including, but not limited to, those involving rational algebraic expressions (e.g., $\frac{1}{a-x} > \frac{x}{x-b}$), trigonometric functions, exponential functions, and the modulus function.

Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation, and simple algebraic division; use of the factor theorem ***and the remainder theorem; use of equating coefficients in identities.***

Know, understand and use the relationship between the roots and coefficients of quadratic equations.

Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear ***and higher degree*** expressions).

Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of linear ***and other*** functions, $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$ ***and other rational functions such as*** $\boldsymbol{y = \frac{x}{(x-a)^2}}$ (including their vertical and horizontal asymptotes); ***behaviour as*** $\boldsymbol{x \to \pm\infty}$; interpret the algebraic solution of equations graphically; use intersection points of graphs to solve equations.

Understand and use proportional relationships and their graphs.

Understand and use composite functions; inverse functions and their graphs.

Understand the effect of simple transformations on the graph of $y = \mathrm{f}(x)$ including sketching associated graphs: $y = a\mathrm{f}(x)$, $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$, $y = \mathrm{f}(ax)$, and combinations of these transformations.

Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than three terms, numerators constant or linear).

Understand what is meant by the limit of a function $\boldsymbol{\mathrm{f}(x)}$ ***as*** $\boldsymbol{x}$ ***tends to a specific value at which the function is undefined, including the notation*** $\boldsymbol{x \to \infty}$***, and be able to find such limits in simple cases.***

Use functions in modelling, including consideration of limitations and refinements of the models.

Coordinate geometry in the (x, y) plane

Know, understand and use the equation of a straight line, including the forms $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular.

Be able to use straight line models in a variety of contexts.

Know, understand and use the coordinate geometry of the circle including using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle; ***know, understand*** and use basic circle theorems:

- ***The angle subtended by an arc at the centre is twice the angle it subtends at the circumference***.
- The angle on the circumference subtended by a diameter is a right angle.
- ***Two angles subtended by a chord in the same segment are equal.***
- A radius or diameter bisects a chord if and only if it is perpendicular to the chord.
- For a point P on the circumference, the radius or diameter through P is perpendicular to the tangent at P.
- ***For a point* P *on the circumference, the angle between the tangent and a chord through* P *equals the angle subtended by the chord in the alternate segment.***
- ***Opposite angles of a cyclic quadrilateral are supplementary.***

Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.

Use parametric equations in modelling in a variety of contexts.

Sequences and series

Know, understand and use the binomial expansion of $(a + bx)^n$ for positive integer n; the notations $n!$ and nCr (***and*** $\boldsymbol{\binom{n}{r}}$ ***and*** ^nC_r) ***and their algebraic definitions***; link to binomial probabilities.

Extend the binomial expansion of $(a + bx)^n$ to any rational n, including its use for approximation; be aware that the expansion is valid (***converges***) for $\left|\frac{bx}{a}\right| < 1$ (proof not required).

Use $\boldsymbol{n!}$ ***and*** ^nC_r ***in the context of permutations and combinations.***

Work with sequences including those given by a formula for the nth term and those generated by a simple relations of the form $x_{n+1} = \mathrm{f}(x_n)$, ***or*** $\boldsymbol{x_{n+1} = \mathbf{f}(x_n, x_{n-1})}$; increasing sequences; decreasing sequences; periodic sequences.

Understand and use sigma notation for sums of series.

Understand and work with arithmetic sequences and series, including ***knowledge of*** the formulae for nth term and the sum to n terms.

Understand and work with geometric sequences and series including ***knowledge of*** the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $|r| < 1$.

Understand what is meant by the limit of a sequence, including the notation $\boldsymbol{x_n \to a}$ ***as*** $\boldsymbol{n \to \infty}$***, and be able to find such a limit in simple cases.***

Use sequences and series in modelling.

Trigonometry

Know, understand and use the definitions of sine, cosine, and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form $\frac{1}{2}ab\sin C$.

Work with radian measure, including use for arc length and area of sector.

Know, understand and use the standard small angle approximations of $\sin\theta$, $\cos\theta$, and $\tan\theta$:

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$ where θ is in radians.

Understand and use the sine, cosine, and tangent functions; their graphs, symmetries, and periodicity.

Know and use exact values of $\sin\theta$ and $\cos\theta$ for

$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and integer multiples.

Know and use exact values of $\tan\theta$ for

$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and (appropriate) integer multiples.

Know, understand and use the definitions of sec, cosec, and cot and of $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$; their relationships to sin, cos, and tan; understand their graphs, their ranges and domains.

Know, understand and use $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Know, understand and use $\sin^2\theta + \cos^2\theta = 1$,

$\sec^2\theta = 1 + \tan^2\theta$, and $\text{cosec}^2\,\theta = 1 + \cot^2\theta$.

Know, understand and use double angle formulae; use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$, and $\tan(A \pm B)$; understand geometrical proofs of these formulae.

Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.

Find general solutions to trigonometric equations, including quadratic equations in sin, cos, or tan and equations involving linear multiples of the unknown angle; for example, $\sin(3x + \frac{\pi}{5}) = \frac{1}{2}$.

Construct proofs involving trigonometric functions and identities.

Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.

Exponentials and logarithms

Know and use the function a^x and its graph, where a is positive.

Know and use the function e^x and its graph.

Know that the gradient of e^{kx} is equal to $k\mathrm{e}^{kx}$, and hence understand why the exponential model is suitable in many applications.

Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive $(a \neq 1)$ and $x > 0$.

Know and use the function $\ln x$ and its graph.

Know and use $\ln x$ as the inverse function of e^x.

Know, understand and use the laws of logarithms:

$\log_a x + \log_a y = \log_a xy$;

$\log_a x - \log_a y = \log_a \frac{x}{y}$;

$k \log_a x = \log_a x^k$

(including, for example, $k = -1$ and $k = -\frac{1}{2}$).

Understand and use the change of base formula for logarithms:

$$\boldsymbol{\log_a x = \frac{\log_b x}{\log_b a}}$$

Solve equations of the form $a^x = b$.

Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.

Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, or exponential growth as a model for population growth); consideration of limitations and refinements of exponential models.

Differentiation

Candidates should have an informal understanding of continuity and differentiability.

Understand and use the derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y = \mathrm{f}(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second ***and higher derivatives***; differentiation from first principles for small positive integer powers of x, and for $\sin x$ and $\cos x$.

Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves and points of inflection.

Differentiate x^n, for rational values of n, and related constant multiples, sums and differences.

Differentiate e^{kx}, a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$ ***and other trigonometric functions*** and related sums, differences and constant multiples.

Know, understand and use the derivative of $\ln x$.

Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, points of inflection.

Identify where functions are increasing or decreasing.

Differentiate using the product rule, the quotient rule, and the chain rule, including problems involving connected rates of change and inverse functions.

Differentiate simple functions and relations defined implicitly or parametrically, for first ***and higher*** derivatives.

Apply the above to curve sketching.

Construct simple differential equations in pure mathematics and in context (contexts may include kinematics, population growth, and modelling the relationship between price and demand).

Integration

Candidates should have an informal understanding of integrability.

Know and use the Fundamental Theorem of Calculus, ***including applications to integration by inspection***.

Integrate x^n (***including*** $\boldsymbol{n = -1}$), and related sums, differences and constant multiples.

Integrate e^{kx}, $\sin kx$, and $\cos kx$, and related sums, differences, and constant multiples.

Evaluate definite integrals; use a definite integral to find the area under a curve and the area between two curves.

Understand and use integration as the limit of a sum.

Carry out simple ***and more complex*** cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively.

(Integration by substitution includes finding a suitable substitution and is ***not*** limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.)

Integrate using partial fractions that are linear ***and repeated linear*** in the denominator.

Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.

(Separation of variables may require factorisation involving a common factor.)

Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

Numerical methods

Locate roots of $\mathrm{f}(x) = 0$ by considering changes of sign of $\mathrm{f}(x)$ in an interval of x on which $\mathrm{f}(x)$ is sufficiently well-behaved.

Understand how change of sign methods can fail.

Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams.

Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = \mathrm{g}(x_n)$.

Understand how such methods can fail.

Understand and use numerical integration of functions; including the use of the trapezium rule, and estimating the approximate area under a curve and limits that it must lie between.

Use numerical methods to solve problems in context.

Vectors

Use vectors in two dimensions and in three dimensions.

Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.

Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.

Understand and use position vectors; calculate the distance between two points represented by position vectors.

Know how to relate the position vector of the point that divides AB in a given ratio to the position vectors of the points A and B.

Use vectors to solve problems in pure mathematics and in context, including forces and kinematics.

Section B: Mechanics and Probability/Statistics

Content

Quantities and units in mechanics

Know, understand and use fundamental quantities and units in the S.I. system: length, time, mass.

Know, understand and use derived quantities and units: velocity, acceleration, force, weight, moment.

Kinematics

Know, understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration.

Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time, and interpretation of gradient; velocity against time, and interpretation of gradient and area under the graph.

Know, understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to 2 dimensions using vectors.

Use calculus in kinematics for motion in a straight line:
$v = \frac{\mathrm{d}r}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$, $r = \int v \,\mathrm{d}t$, $v = \int a \,\mathrm{d}t$; extend to 2 dimensions using vectors.

Model motion under gravity in a vertical plane using vectors; projectiles.

Forces and Newton's laws

Understand the concept of a force; understand and use Newton's first law.

Know, understand and use Newton's second law for motion in a straight line, including situations where forces need to be resolved (in 2 ***or 3*** dimensions); application to problems involving smooth pulleys and connected particles.

Understand and use weight, and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy.

(The inverse square law for gravitation is not required and g may be assumed to be constant, but students should be aware that g is not a universal constant but depends on location.)

Know, understand and use Newton's third law; resolving forces in 2 ***or 3*** dimensions; equilibrium of a particle under forces in 2 ***or 3*** dimensions.

Understand and use addition of forces; resultant forces; dynamics for motion in a plane.

Know, understand and use the $F \leq \mu R$ model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.

Moments

Understand and use moments in simple static contexts; ***questions will <u>not</u> be restricted to those involving forces in two perpendicular directions. Questions may be set on equilibria of rigid bodies, including bodies in contact, and on breaking of equilibrium, for example by toppling or slipping.***

Understand and use the idea of centre of mass; the position of the centre of mass of any shapes used will either be given or deducible by the use of symmetry.

Statistical sampling

Understand and use the terms 'population', 'sample' and ***'random sample'***.

Data presentation and interpretation

Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency, and connect to probability distributions.

Interpret measures of central tendency and variation, extending to standard deviation.

Be able to calculate standard deviation, including from summary statistics.

Probability

Understand and use mutually exclusive, independent, ***and complementary*** events when calculating probabilities. Link to discrete and continuous distributions.

Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables.

Know, understand and use the formula:

$$\mathbf{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

Know, understand and use the conditional probability formula:

$$\mathrm{P(A|B)} = \frac{\mathrm{P(A\cap B)}}{\mathrm{P(B)}}$$

Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.

Use combinatorial arguments, including the use of $n!$ and ${}^nC_r$, in the context of calculating probabilities.

Statistical distributions

Understand and use simple, discrete probability distributions (calculation of expectation and variance of discrete random variables is ***included***), including the Binomial distribution as a model; calculate probabilities using the Binomial distribution.

The discrete uniform distribution as a model; calculate probabilities using the discrete uniform distribution.

Understand and use the Normal distribution; find probabilities using the Normal distribution; ***convert to the standard Normal distribution by translation and scaling.***

Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the Binomial or Normal model may not be appropriate.

Statistical hypothesis testing

Understand and apply the language of statistical hypothesis testing, developed through a Binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tailed test, 2-tailed test, critical value, critical region, acceptance region, p-value.

Conduct a statistical hypothesis test for the proportion in the Binomial distribution and interpret the results in context.

Understand that a sample is being used to make an inference about the population, and appreciate how the significance level and the probability of incorrectly rejecting the null hypothesis are related.

Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given, or assumed variance and interpret the results in context.

MATHEMATICS 2

The specification for Mathematics 2 assumes full knowledge and understanding of the relevant parts of the specification for Mathematics 1 as set out in the introduction. Candidates should be aware that questions in this paper may be set on any relevant parts of the Mathematics 1 or Mathematics 2 specifications.

Section A: Pure Mathematics

	Content
Complex numbers	
	Solve any quadratic equation with real ***or complex*** coefficients; solve cubic or quartic equations with real ***or complex*** coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).
	Add, subtract, multiply, and divide complex numbers in the form $x+\mathrm{i}y$ with x and y real; understand and use the terms 'real part' and 'imaginary part'.
	Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.
	Use and interpret Argand diagrams.
	Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed).
	Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed).
	Construct and interpret simple loci in the Argand diagram such as, ***but not limited to***, $\lvert z-a\rvert = r$ and $\arg(z-a) = \theta$ (knowledge of radians is assumed).

Matrices

Add, subtract, and multiply conformable matrices; multiply a matrix by a scalar.

Understand and use zero and identity matrices.

Use matrices to represent linear transformations in 2-D; successive transformations; single transformations in 3-D (3-D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes). (Knowledge of 3-D vectors is assumed.)

Find invariant points and lines for a linear transformation.

Calculate determinants of 2×2 matrices ***and interpret as scale factors, including the effect on orientation***.

Understand and use singular and non-singular matrices; properties of inverse matrices.

Calculate and use the inverse of a non-singular 2×2 matrix.

Further algebra and functions

Understand and use the relationship between roots and coefficients of polynomial equations up to quartic ***and higher degree*** equations.

Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).

Know and use partial fractions in which the denominator may include quadratic factors of the form $ax^2 + c$ for $c > 0$, and in which the degree of the numerator may be equal to, or exceed, the degree of the denominator.

Understand and use the method of differences for summation of series, including the use of partial fractions.

Recognise and use the series expansion of e^x.

Sketch curves of the form $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$; find equations of their asymptotes where appropriate.

Further calculus

Differentiate inverse trigonometric functions.

Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.

Integrate functions of the form $\left(1 + x^2\right)^{-1}$ ***and*** $\left(1 - x^2\right)^{-\frac{1}{2}}$ ***and be able to choose trigonometric substitutions to integrate associated functions.***

Integrate using partial fractions (including those with quadratic factors in the denominator).

Integrate using reduction formulae.

Further vectors

Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.

Understand and use the scalar product of two vectors, including geometrical interpretation and formal algebraic manipulation; for example, $\mathbf{a}.(\mathbf{b} + \mathbf{c}) = \mathbf{a}.\mathbf{b} + \mathbf{a}.\mathbf{c}$

Section B: Mechanics

Content

Energy, work, and power

Understand and use the concepts of energy (kinetic and potential), work, and power.

Understand and use the principle of conservation of energy.

Collisions

Understand the mechanics of collisions in simple situations.

Understand and use the principle of conservation of momentum and, when appropriate, the conservation of energy applied to collisions.

Understand and use the coefficient of restitution (e) for collisions, including the special cases $e = 1$ and $e = 0$.

Questions involving successive impacts may be set.

Knowledge of oblique impacts will not be required.

Hooke's law

Know, understand and use Hooke's law for strings and springs, including the formula $T = kx = \frac{\lambda x}{l}$ where k is the stiffness and λ is the modulus of elasticity.

Understand and use elastic potential energy, including knowledge of the formula $E = \frac{1}{2}kx^2 = \frac{\lambda x^2}{2l}$.

Section C: Probability/Statistics

Content

Probability distributions

Know, understand and use the Poisson distribution; find probabilities using the Poisson distribution.

Calculate the mean and variance of the Poisson distribution.

Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the Binomial, the Normal or the Poisson model may not be appropriate.

Use the Poisson distribution as an approximation to the Binomial distribution and know under what conditions this is appropriate.

Use the Normal distribution as an approximation to the Binomial distribution or the Poisson distribution and know under what conditions these are appropriate.

Know, understand and use the continuous uniform distribution; find probabilities using the continuous uniform distribution.

Calculate the mean and variance of the continuous uniform distribution.

Understand and use the mathematics of continuous probability density functions and cumulative distribution functions; including finding probabilities and the calculation of mean, variance, median, mode, and expectation by explicit integration for a given (possibly unfamiliar) distribution; the notation $\mathrm{f}(x) = \mathrm{F}'(x)$.

MATHEMATICS 3

The specification for Mathematics 3 assumes full knowledge and understanding of the relevant parts of the specification for Mathematics 1 and Mathematics 2 as set out in the introduction. Candidates should be aware that questions in this paper may be set on any relevant parts of the Mathematics 1, Mathematics 2 or Mathematics 3 specifications.

Section A: Pure Mathematics

	Content
Further Complex numbers	
	Know and understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.
	Know and use the definition $\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$ and the form $z = r\mathrm{e}^{\mathrm{i}\theta}$
	Find the n distinct nth roots of $r\mathrm{e}^{\mathrm{i}\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon in the Argand diagram.
	Use complex numbers, including complex roots of unity, to solve geometric problems.
Further Matrices	
	Calculate determinants of 3×3 matrices and interpret as scale factors, including the effect on orientation.
	Calculate and use the inverse of non-singular 3×3 matrices.
	Solve three linear simultaneous equations in three variables by use of the inverse matrix.
	Interpret geometrically the solution and failure of solution of three simultaneous linear equations.
Further algebra and functions	
	Find the Maclaurin series of a function including the general term.
	Know and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$, and $(1+x)^n$, and be aware of the range of values of x for which they are valid (proof not required).

Further calculus

Calculate lengths of curves in Cartesian or parameterised Cartesian coordinates.

Derive formulae for and calculate volumes of revolution.

Understand and evaluate the mean value of a function.

Further vectors

Understand and use the vector and Cartesian forms of the equation of a plane.

Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two planes and the angle between a line and a plane.

Find the intersection of a line and a plane.

Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

Understand and use the vector product, including the geometrical interpretation; use vector products to calculate the area of a triangle or parallelogram and to determine whether vectors are parallel.

Understand and use the equation of a line in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$

Polar coordinates

Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.

(It will be assumed that $r \geq 0$***; the range of*** θ ***will be given if appropriate.)***

Sketch curves with r given as a function of θ, including the use of trigonometric functions.

Find the area enclosed by a polar curve.

Hyperbolic functions

Know, understand and use the definitions of hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$, $\mathbf{sech}\,\boldsymbol{x}$, $\mathbf{cosech}\,\boldsymbol{x}$, $\mathbf{coth}\,\boldsymbol{x}$ including their domains and ranges, and be able to sketch their graphs.

Know, understand and use standard formulae for algebraic relations between hyperbolic functions, such as $\mathbf{cosh^2}\boldsymbol{x} - \mathbf{sinh^2}\boldsymbol{x} = \mathbf{1}$.

Differentiate and integrate hyperbolic functions.

Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.

Derive and use the logarithmic forms of the inverse hyperbolic functions.

Integrate functions of the form $\left(x^2+1\right)^{-\frac{1}{2}}$ and $\left(x^2-1\right)^{-\frac{1}{2}}$ and be able to choose substitutions to integrate associated functions.

Differential equations

Find and use an integrating factor to solve differential equations of the form $\frac{dy}{dx} + P(x)y = Q(x)$; recognise when it is appropriate to do so.

Find both general and particular solutions to differential equations, ***including by methods that will be indicated if appropriate.***

Use differential equations in modelling in kinematics and in other contexts.

Solve differential equations of the form $y'' + ay' + by = 0$, where a and b are constants, by using the auxiliary equation.

Know, understand and use the form of the solution of the differential equations in cases when the discriminant of the auxiliary equation is positive, zero, or negative.

Solve differential equations of the form $y'' + ay' + by = f(x)$ where a and b are constants, by solving the homogeneous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential, or trigonometric function).

Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and relate the solution to the motion, ***and understand the implications in physical situations.***

Model damped oscillations using second order differential equations and interpret their solutions.

Analyse models of situations with one independent variable and two dependent variables as a pair of coupled first order simultaneous equations and be able to solve them, for example predator-prey models.

Use given substitutions to transform differential equations.

Section B: Mechanics

Content

Further collisions

Understand and be able to use the concept of impulse.

Analyse collisions involving oblique impacts, including the use of the coefficient of restitution. Questions involving successive impacts may be set.

Centre of mass

Understand and be able to apply the principle that the effect of gravity is equivalent to a single force acting at the body's centre of mass.

Find the position of the centre of mass of a uniform rigid body using symmetry.

Determine the centre of mass of a system of particles or the centre of mass of a composite rigid body.

Use integration to determine the position of the centre of mass of a uniform lamina or a uniform solid of revolution.

Circular motion

Know, understand and use the definitions of angular velocity, velocity, speed, and acceleration in relation to a particle moving in a circular path with constant speed; includes the use of both ω and $\dot{\theta}$.

Know, understand and use the relationships $v = r\omega$ and $a = \frac{v^2}{r} = r\omega^2 = v\omega$ for motion in a circle with constant speed.

Analyse motion with variable speed on an arc of a circle, including motion in a vertical circle.

Moments of inertia will <u>not</u> be examined.

Differential equations

Use differential equations to analyse models of particles moving under the action of variable forces, where forces will not necessarily be given as a function of time; including recognising when it is appropriate to use $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$

Solve the equation for simple harmonic motion $\ddot{x} = -\omega^2 x$ and understand applications to physical situations, including the approximate simple harmonic motion of a pendulum.

Section C: Probability/Statistics

Content

Independent random variables

Understand and use the idea of independent random variables.

Algebra of expectation

Know, understand and use the algebra of expectation:

$\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$

$\mathrm{Var}(X) = \mathrm{E}(X^2) - \mathrm{E}(X)^2$

$\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

and for independent random variables:

$\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$

Knowledge of the relation

$\mathrm{Var}(aX \pm bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) \pm 2ab\,\mathrm{Cov}(X, Y)$

will not be required.

Use cumulative distribution functions to calculate the probability density function of a related random variable; for example, X^2 from X.

Knowledge of generating functions will not be required.

STEP MATHEMATICS 2019

Notation and Required Formulae

Introduction

The **notation** for STEP follows the notation for the A level examinations[1] with some minor additions and omissions. STEP papers are set in LaTeX fonts, which are not the same as the usual fonts used for A level.

The **required formulae** for each STEP paper are the formulae that candidates must be able to use without them being provided. If other formulae are required for a particular question, they will be given in the question (or candidates will be asked to derive them); there are no Formulae Booklets for STEP examinations.

The required formulae are those required for the corresponding AS or A level as set out in the Department for Education's guidance documents,[1] with some additions and omissions. Nearly all the additions can be found in the AS or A level Formulae Booklets provided by the individual examination boards, but candidates are not expected to know all the formulae in these booklets. Throughout the tables that follow, notation and formulae that do not appear in the Department for Education's corresponding guidance documents are indicated by a '•' in the 'Papers' column.

The formulae are usually given in their simplest forms. For example, the derivative of $\sin x$ rather than $\sin kx$ is given, the latter being easily derivable from the former.

Some formulae are omitted because it is better not to learn them. For example, the derivative of $\sin^{-1} x$ is included, but the derivative of $\cos^{-1} x$ is not; it is better to understand that (for acute angles) $\cos^{-1} x = \frac{1}{2}\pi - \sin^{-1} x$ so that the only difference in the derivatives is a minus sign.

[1] See https://www.gov.uk/government/publications/gce-as-and-a-level-mathematics and https://www.gov.uk/government/publications/gce-as-and-a-level-further-mathematics

NOTATION

Set notation			
Notation	**Meaning**	**Comment**	**Papers**
$\in$	is an element of		1, 2, 3
$\notin$	is not an element of		1, 2, 3
$\subseteq$	is a subset of		1, 2, 3
$\subset$	is a proper subset of		1, 2, 3
$\{x_1, x_2, \ldots, x_n\}$	the set with elements x_1, x_2, $\ldots$, x_n		1, 2, 3
$\varnothing$	the empty set	only used with explanation	1, 2, 3
A'	the complement set of the set A	only used with explanation	1, 2, 3
$\mathbb{N}$	the set of natural numbers $\{1, 2, 3, \ldots\}$		1, 2, 3
$\mathbb{Z}$	the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$		1, 2, 3
$\mathbb{Q}$	the set of rational numbers $\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$		1, 2, 3
$\mathbb{R}$	the set of real numbers		1, 2, 3
$\mathbb{C}$	the set of complex numbers		2, 3
$\cup$	union of sets		1, 2, 3
$\cap$	intersection of sets		1, 2, 3
(x, y)	the ordered pair x, y	for example, coordinates	1, 2, 3

Miscellaneous symbols			
Notation	**Meaning**	**Comment**	**Papers**
$=$	is equal to		1, 2, 3
$\neq$	is not equal to		1, 2, 3
$\equiv$	is identical to, or is equivalent to, or is congruent to		1, 2, 3
$\approx$	is approximately equal to		1, 2, 3
∞	infinity		1, 2, 3
$\propto$	is proportional to		1, 2, 3
$<$	is less than		1, 2, 3
$\leqslant$	is less than or equal to		1, 2, 3
$>$	is greater than		1, 2, 3
$\geqslant$	is greater than or equal to		1, 2, 3
$\therefore$	therefore		1, 2, 3
$p \Rightarrow q$	p implies q (if p then q)		1, 2, 3
$p \Leftarrow q$	p is implied by q (if q then p)		1, 2, 3
$p \Leftrightarrow q$	p is equivalent to q (p if and only if q)		1, 2, 3
S_n	the sum to n terms of a progression		1, 2, 3
S_∞	the sum to infinity of a progression		1, 2, 3
$x \longrightarrow \infty$	x tends to ∞		1, 2, 3
$x_n \longrightarrow a$	x_n tends to a	for sequences when $n \longrightarrow \infty$	1, 2, 3

Operations			
Notation	**Meaning**	**Comment**	**Papers**
$a \pm b$	a plus or minus b		1, 2, 3 •
$a \mp b$	a minus or plus b		1, 2, 3 •
$a \times b$, ab, $a.b$	a multiplied by b	$a.b$ not usually used	1, 2, 3
$a \div b$, $\frac{a}{b}$, a/b	a divided by b	a/b is not given in the A level notation list; $a \div b$ not usually used	1, 2, 3 •
$\sum_{i=m}^{n} a_i$	$a_m + a_{m+1} + \cdots + a_n$	only the case $m = 1$ is given in the A level notation list	1, 2, 3 •
$\prod_{i=m}^{n} a_i$	$a_m a_{m+1} \cdots a_n$	only the case $m = 1$ is given in the A level notation list	1, 2, 3 •
$\sqrt{a}$	the positive square root of a	$a \in \mathbb{R}$, $a \geqslant 0$	1, 2, 3
$\lvert a \rvert$	the modulus of a	$a \in \mathbb{R}$	1, 2, 3
$n!$	n factorial, $n \in \mathbb{N}$	$0! = 1$, by definition	1, 2, 3
$\binom{n}{r}$ or ${}^nC_r$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$		1, 2, 3

Functions			
Notation	**Meaning**	**Comment**	**Papers**
$\lim\limits_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a		1, 2, 3
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the first derivative of y with respect to x		1, 2, 3
$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the nth derivative of y with respect to x		1, 2, 3
$\dot{x}$ and $\ddot{x}$	the first and second derivatives of x with respect to t	where t is time, unless otherwise specified	1, 2, 3
$\mathrm{f}'(x)$	the first derivative of f evaluated at x		1, 2, 3
$\mathrm{f}''(x)$	the second derivative of f evaluated at x		1, 2, 3
$\mathrm{f}^{(n)}(x)$	the nth derivative of f evaluated at x		1, 2, 3
$\int \mathrm{f}(x)\,\mathrm{d}x$	the indefinite integral of $\mathrm{f}(x)$ with respect to x		1, 2, 3
$\int_a^b \mathrm{f}(x)\,\mathrm{d}x$	the definite integral of $\mathrm{f}(x)$ with respect to x between the limits of $x = a$ and $x = b$		1, 2, 3
e	base of natural logarithms		1, 2, 3
e^x, $\exp x$	exponential function of x		1, 2, 3
$\log_a x$	logarithm to base a of x		1, 2, 3
$\ln x$, $\log_\mathrm{e} x$	natural logarithm of x		1, 2, 3
sin, cos, tan, cosec, sec, cot	the trigonometric functions		1, 2, 3
$\sin^{-1}$, etc	the inverse trigonometric functions	arcsin, etc, will not be used	1, 2, 3
sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions		3
$\sinh^{-1}$, etc	the inverse hyperbolic functions	arsinh, etc, will not be used	3

Complex numbers			
Notation	**Meaning**	**Comment**	**Papers**
i	square root of -1	j will not be used	2, 3
$x+\mathrm{i}y$	complex number with real part x and imaginary part y		2, 3
$r(\cos\theta+\mathrm{i}\sin\theta)$	complex number with modulus r and argument θ	$r \geqslant 0$; the range of θ will be given if required	2, 3
$\mathrm{Re}(z)$	the real part of z	$\mathrm{Re}(z)=x$ if $z=x+\mathrm{i}y$	2, 3
$\mathrm{Im}(z)$	the imaginary part of z	$\mathrm{Im}(z)=y$ if $z=x+\mathrm{i}y$	2, 3
$\arg(z)$	the argument of z	$\arg(z)=\theta$ if $z=r(\cos\theta+\mathrm{i}\sin\theta)$; the range of $\arg(z)$ will be given if required	2, 3
$\lvert z\rvert$	the modulus of z	$\lvert z\rvert=r$ if $z=r(\cos\theta+\mathrm{i}\sin\theta)$	2, 3
z^*	the complex conjugate of z	$z^*=x-\mathrm{i}y$ if $z=x+\mathrm{i}y$	2, 3

Matrices			
Notation	**Meaning**	**Comment**	**Papers**
$\mathbf{M}$	the matrix $\mathbf{M}$		2, 3
M_{ij}	the entry in the ith row and jth column of the matrix $\mathbf{M}$	only used with explanation	2, 3 •
$\mathbf{0}$	matrix with all entries 0		2, 3
$\mathbf{I}$	identity matrix		2, 3
$\mathbf{M}^{-1}$	the inverse of the (square) matrix $\mathbf{M}$		2, 3
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$		2, 3
$\det \mathbf{M}$	determinant of the (square) matrix $\mathbf{M}$		2, 3
$\mathbf{Mr}$	image of the column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$		2, 3

Vectors			
Notation	**Meaning**	**Comment**	**Papers**
$\mathbf{a}$	the vector $\mathbf{a}$		1, 2, 3
$\overrightarrow{AB}$	the vector represented by the directed line segment AB		1, 2, 3
$\widehat{\mathbf{a}}$	the unit vector in the direction of $\mathbf{a}$	only used with explanation	1, 2, 3
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the Cartesian axes		1, 2, 3
$\lvert\mathbf{a}\rvert$	the magnitude of $\mathbf{a}$		1, 2, 3
$\lvert\overrightarrow{AB}\rvert$	the magnitude of $\overrightarrow{AB}$		1, 2, 3
$\mathbf{r}$	position vector		1, 2, 3
$\mathbf{s}$	displacement vector		1, 2, 3
$\mathbf{a} \,.\, \mathbf{b}$	the scalar product of vectors $\mathbf{a}$ and $\mathbf{b}$		2, 3
$\mathbf{a} \times \mathbf{b}$	the vector product of vectors $\mathbf{a}$ and $\mathbf{b}$		3 •

Probability/Statistics			
Notation	**Meaning**	**Comment**	**Papers**
A, B, C, etc	events		1, 2, 3
$A \cup B$	union of events A and B		1, 2, 3
$A \cap B$	intersection of events A and B		1, 2, 3
$\mathrm{P}(A)$	probability of the event A		1, 2, 3
A'	complement of event A	only used with explanation	1, 2, 3
$\mathrm{P}(A \mid B)$	probability of the event A conditional on (i.e. given) the event B		1, 2, 3
x, y, r, etc	values of the random variables X, Y, R, etc		1, 2, 3
$\mathrm{P}(X = x)$	probability function of a discrete random variable X		1, 2, 3
$\mathrm{f}(x)$	probability density function (p.d.f.) of a continuous random variable		2, 3
$\mathrm{F}(x)$	cumulative distribution function (c.d.f.) of a continuous random variable		1, 2, 3
$\mathrm{E}(X)$	expectation of the random variable X		1, 2, 3
$\mathrm{Var}\,(X)$	variance of the random variable X		1, 2, 3
$\sim$	has the distribution		1, 2, 3
$\mathrm{B}(n, p)$	Binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in any trial	$q = 1 - p$	1, 2, 3
$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2		1, 2, 3
$\mathrm{N}(0, 1)$	the standard Normal distribution		1, 2, 3
ϕ, Φ	probability density function and cumulative distribution function of a random variable with standard Normal distribution	knowledge of formulae is not required; only used with explanation	1, 2, 3

Mechanics			
Notation	**Meaning**	**Comment**	**Papers**
kg	kilogram		1, 2, 3
m	metre		1, 2, 3
km	kilometre		1, 2, 3
$\mathrm{m\,s^{-1}}$	metres per second		1, 2, 3
$\mathrm{m\,s^{-2}}$	metres per second per second	acceleration	1, 2, 3
N	newton		1, 2, 3
N m	newton metre	moment of a force, for example	1, 2, 3
J	joule		1, 2, 3
t	time		1, 2, 3
s	displacement		1, 2, 3
u	initial speed		1, 2, 3
v	speed or final speed		1, 2, 3
a	acceleration		1, 2, 3
g	acceleration due to gravity		1, 2, 3
μ	coefficient of friction		1, 2, 3
e	coefficient of restitution		2, 3
k	stiffness		2, 3
λ	modulus of elasticity		2, 3
ω	angular speed		3

REQUIRED FORMULAE

Roots of polynomials		
Formula	**Comment**	**Papers**
$ax^2 + bx + c = 0$ has roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		1, 2, 3
For $ax^2 + bx + c = 0$ with roots α and β: $\alpha + \beta = -b/a,\ \ \alpha\beta = c/a$		1, 2, 3
For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ: $\alpha + \beta + \gamma = -b/a,\ \ \alpha\beta + \beta\gamma + \gamma\alpha = c/a,\ \ \alpha\beta\gamma = -d/a$	The pattern is the same for polynomial equations of higher degree	2, 3

Laws of indices		
Formula	**Comment**	**Papers**
$a^x a^y = a^{x+y}$		1, 2, 3
$a^0 = 1$	$a \neq 0$	1, 2, 3
$(a^x)^y = a^{xy}$		1, 2, 3
$a^x = \mathrm{e}^{x \ln a}$	defines a^x when x is not an integer	1, 2, 3 •

Laws of logarithms		
Formula	**Comment**	**Papers**
$x = a^n \Leftrightarrow n = \log_a x$	$x > 0$, $a > 0$ $(a \neq 1)$	1, 2, 3
$\log_a x + \log_a y = \log_a(xy)$		1, 2, 3
$\log_a x - \log_a y = \log_a(x/y)$		1, 2, 3
$k \log_a x = \log_a x^k$	for $x > 0$	1, 2, 3

Sequences and series

Formula	**Comment**	**Papers**
General (nth) term of an arithmetic progression: $u_n = a + (n-1)d$	d is the common difference	1, 2, 3
General (nth) term of a geometric progression: $u_n = ar^{n-1}$	r is the common ratio	1, 2, 3
Sum of an arithmetic progression: $S_n = \frac{1}{2}n\{2a + (n-1)d\}$	or: $S_n = an + \frac{1}{2}n(n-1)d$	1, 2, 3 •
Sum of a geometric progression: $S_n = \dfrac{a(1-r^n)}{1-r}$		1, 2, 3 •
Sum to infinity of a geometric progression: $S_\infty = \dfrac{a}{1-r}$	$\lvert r\rvert < 1$	1, 2, 3 •
${}^nC_r = \dfrac{n!}{(n-r)!r!}$		1, 2, 3 •
$(a+b)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$	Binomial expansion, $n \in \mathbb{N}$	1, 2, 3 •
$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \cdots + \frac{k(k-1)\cdots(k-r+1)}{r!}x^r + \cdots$	$\lvert x\rvert < 1,\ k \in \mathbb{Q}$	1, 2, 3 •
$\displaystyle\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$		1, 2, 3 •
$\mathrm{f}(x) = \displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}\,\mathrm{f}^{(r)}(0)\,x^r$	Maclaurin series	3 •
$\mathrm{e}^x = \displaystyle\sum_{r=0}^{\infty} \frac{x^r}{r!}$	converges for all x	2, 3 •
$\ln(1+x) = \displaystyle\sum_{r=1}^{\infty} (-1)^{r+1}\frac{x^r}{r}$	converges for $-1 < x \leqslant 1$	3 •
$\sin x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{(2r+1)!}$	converges for all x	3 •
$\cos x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r}}{(2r)!}$	converges for all x	3 •

Coordinate geometry		
Formula	**Comment**	**Papers**
The straight line graph with gradient m passing through the point (x_1, y_1) has equation $y - y_1 = m(x - x_1)$		1, 2, 3
Straight lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$		1, 2, 3

Trigonometry		
Formula	**Comment**	**Papers**
Sine rule for the triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$		1, 2, 3
Cosine rule in the triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$		1, 2, 3
Area of triangle ABC: $\frac{1}{2}ab\sin C$		1, 2, 3
$\cos^2 A + \sin^2 A = 1$		1, 2, 3
$\sec^2 A = 1 + \tan^2 A$		1, 2, 3
$\operatorname{cosec}^2 A = 1 + \cot^2 A$		1, 2, 3
$\sin 2A = 2\sin A\cos A$		1, 2, 3
$\cos 2A = \cos^2 A - \sin^2 A$		1, 2, 3
$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$	$A \neq (k + \frac{1}{2})\frac{\pi}{2}$, $k \in \mathbb{Z}$	1, 2, 3
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$		1, 2, 3
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$		1, 2, 3
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$A \pm B \neq (k + \frac{1}{2})\pi$, $k \in \mathbb{Z}$	1, 2, 3 •
$\sin\theta \approx \theta\,, \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2\,, \quad \tan\theta \approx \theta$	θ small (compared with 1); θ in radians	1, 2, 3 •

Hyperbolic functions		
Formula	**Comment**	**Papers**
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	by definition	3
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	by definition	3
$\tanh x = \dfrac{\sinh x}{\cosh x}$	by definition	3
$\cosh^2 A - \sinh^2 A = 1$		3 •
$\text{sech}^2 A = 1 - \tanh^2 A$		3 •
$\text{cosech}^2 A = \coth^2 A - 1$		3 •
$\sinh 2A = 2 \sinh A \cosh A$		3 •
$\cosh 2A = \cosh^2 A + \sinh^2 A$		3 •
$\tanh 2A = \dfrac{2 \tanh A}{1 + \tanh^2 A}$		3 •
$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$		3 •
$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$		3 •
$\tanh(A \pm B) = \dfrac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$		3 •

Derivatives			
Function	**Derivative**	**Comment**	**Papers**
$\sin x$	$\cos x$		1, 2, 3
$\cos x$	$-\sin x$		1, 2, 3
$\tan x$	$\sec^2 x$		1, 2, 3 •
$\cot x$	$-\operatorname{cosec}^2 x$		1, 2, 3 •
$\sec x$	$\sec x \tan x$		1, 2, 3 •
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$		1, 2, 3 •
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1-x^2}}$		2, 3 •
$\tan^{-1} x$	$\dfrac{1}{1+x^2}$		2, 3 •
$\sinh x$	$\cosh x$		3
$\cosh x$	$\sinh x$		3
$\tanh x$	$\operatorname{sech}^2 x$		3 •
$\coth x$	$-\operatorname{cosech}^2 x$		3 •
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$		3 •
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1+x^2}}$		3 •
$\tanh^{-1} x$	$\dfrac{1}{1-x^2}$		3 •
e^x	e^x		1, 2, 3
$\ln x$	$\dfrac{1}{x}$		1, 2, 3
$f(x)+g(x)$	$f'(x)+g'(x)$		1, 2, 3
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$	product rule	1, 2, 3
$\dfrac{f(x)}{g(x)}$	$\dfrac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	quotient rule	1, 2, 3 •
$f\left(g(x)\right)$	$f'\left(g(x)\right)g'(x)$	chain rule	1, 2, 3

Integrals			
Function	**Integral**	**Comment**	**Papers**
x^n	$\frac{1}{n+1}x^{n+1}+c$	$n \neq -1$	1, 2, 3
x^{-1}	$\ln\lvert x\rvert+c$		1, 2, 3
$\cos x$	$\sin x+c$		1, 2, 3
$\sin x$	$-\cos x+c$		1, 2, 3
$\sinh x$	$\cosh x+c$		3
$\cosh x$	$\sinh x+c$		3
$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1}x+c$	$-1<x<1$	2, 3 •
$\dfrac{1}{1+x^2}$	$\tan^{-1}x+c$		2, 3 •
e^x	e^x+c		1, 2, 3
$f'(x)+g'(x)$	$f(x)+g(x)+c$		1, 2, 3
$f'\big(g(x)\big)\,g'(x)$	$f\big(g(x)\big)+c$		1, 2, 3
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert+c$		1, 2, 3 •
$\big(f(x)\big)^n f'(x)$	$\frac{1}{n+1}\big(f(x)\big)^{n+1}+c$	$n \neq -1$	1, 2, 3 •
$u\dfrac{dv}{dx}$	$uv-\displaystyle\int v\frac{du}{dx}\,dx$	integration by parts	1, 2, 3 •

General calculus		
Formula	**Comment**	**Papers**
$\mathrm{f}'(x) = \lim_{h \to 0} \dfrac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	first principles definition	1, 2, 3 •
$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \Big/ \dfrac{\mathrm{d}x}{\mathrm{d}t}$	for the parameterised curve $y = y(t)$, $x = x(t)$	1, 2, 3 •
Area under the curve $y = \mathrm{f}(x)$ and above the x-axis: $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$		1, 2, 3
Volume of revolution about the x-axis: $\pi \int_a^b \left(\mathrm{f}(x)\right)^2 \mathrm{d}x$		3
$\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h(y_0 + y_n) + h(y_1 + y_2 + \cdots + y_{n-1})$	$h = \dfrac{b-a}{n}$, $y_r = y(a + rh)$, trapezium rule	1, 2, 3 •
$\ddot{x} = -\omega^2 x \Rightarrow x = R\sin(\omega t + \alpha)$ or $x = R\cos(\omega t + \beta)$ or $x = A\cos\omega t + B\sin\omega t$	simple harmonic motion	3 •

Circles		
Formula	**Comment**	**Papers**
Length of an arc of a circle of radius r: $r\theta$	θ is angle subtended in radians	1, 2, 3
Area of a sector of a circle of radius r: $\frac{1}{2}r^2\theta$	θ is angle subtended in radians	1, 2, 3

Complex numbers		
Formula	**Comment**	**Papers**
$e^{i\theta} = \cos\theta + i\sin\theta$		3
$z = r(\cos\theta + i\sin\theta) \Rightarrow z^n = r^n(\cos n\theta + i\sin n\theta)$	de Moivre's theorem	3 •
$z^n = 1$ has roots $z = e^{2\pi k i/n}$ where $k = 0, 1, \ldots, (n-1)$	Roots of unity	3 •
Half line with end-point a: $\arg(z-a) = \theta$	θ is the angle between the line and a line parallel to the positive real axis	2, 3
Circle, centre a and radius r: $\lvert z-a\rvert = r$		2, 3

Vectors		
Formula	**Comment**	**Papers**
$\lvert x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\rvert = \sqrt{x^2+y^2+z^2}$		1, 2, 3
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$	scalar product, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$	2, 3
$\mathbf{a}\times\mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	vector product	3 •
$\lvert\mathbf{a}\times\mathbf{b}\rvert = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert$	θ is the acute angle between the vectors	3 •
Equation of the line through the point with position vector $\mathbf{a}$ parallel to $\mathbf{b}$: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$		2, 3
Equation of the plane containing the point with position vector $\mathbf{a}$ and with normal $\mathbf{n}$: $(\mathbf{r}-\mathbf{a}).\mathbf{n} = 0$		3

Matrices		
Formula	**Comment**	**Papers**
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det \mathbf{A} = ad - bc$		2, 3
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \dfrac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	$\det \mathbf{A} \neq 0$	2, 3
AB is equivalent to **B** then **A**	for transformations represented by these matrices	2, 3
$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	$\det \mathbf{AB} \neq 0$	2, 3
$\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$	reflection in the line $y = \pm x$	2, 3 •
$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$	rotation by θ about the z-axis; the direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation	2, 3 •
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	reflection in the plane $z = 0$	2, 3 •

Mechanics		
Formula	**Comment**	**Papers**
mg	weight	1, 2, 3
$F \leqslant \mu R$	frictional force related to normal reaction R	1, 2, 3
$F = ma$	scalar version of Newton's second law; constant mass	1, 2, 3
$\mathbf{F} = m\mathbf{a}$	vector version of Newton's second law; constant mass	1, 2, 3
$\frac{1}{2}mv^2$	kinetic energy	2, 3 •
mgh	change in gravitational potential energy; h is vertical height	2, 3 •
mv	momentum	2, 3 •
$mv - mu$	impulse	2, 3 •
$T = \dfrac{\lambda x}{l} = kx$	Hooke's law	2, 3 •
$E = \dfrac{\lambda x^2}{2l} = \dfrac{1}{2}kx^2$	elastic potential energy	2, 3 •
$v = \dfrac{\mathrm{d}r}{\mathrm{d}t},\quad a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$	motion in a straight line (where acceleration, a, may not be constant)	1, 2, 3 •
$v = u + at,\;\; s = ut + \frac{1}{2}at^2,\;\; s = \frac{1}{2}(u+v)t,\;\; v^2 - u^2 = 2as$	motion in a straight line with constant acceleration, a	1, 2, 3 •
$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\;\; \mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	motion in two (STEP 1) or three (STEP 3) dimensions where acceleration, $\mathbf{a}$, may not be constant)	1, 2, 3 •
$\mathbf{v} = \mathbf{u} + \mathbf{a}t,\;\; \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2,\;\; \mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t,\;\; \mathbf{v}.\mathbf{v} - \mathbf{u}.\mathbf{u} = 2\mathbf{a}.\mathbf{s}$	motion in two (STEP 1) or three (STEP 2) dimensions with constant acceleration, $\mathbf{a}$	1, 2, 3 •
$v_1 - v_2 = -e(u_1 - u_2)$ or relative speed of separation $= e \times$ relative speed of approach	Newton's experimental law	2, 3 •
speed $= r\dot{\theta}$, radial acceleration $= \dfrac{v^2}{r} = r\dot{\theta}^2$ towards the centre, tangential acceleration $= r\ddot{\theta}$	motion in a circle of radius r	3 •

Probability/Statistics		
Formula	**Comment**	**Papers**
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$	probability of the union of two events	1, 2, 3 •
$\mathrm{P}(A \cap B) = \mathrm{P}(A \mid B)\,\mathrm{P}(B)$	probability of the intersection of two events	1, 2, 3 •
$\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$	algebra of expectation	3 •
$\mathrm{Var}\,(aX + bY + c) = a^2\,\mathrm{Var}\,(X) + b^2\,\mathrm{Var}\,(Y)$	algebra of variances for independent variables	3 •
$\mu = \mathrm{E}(X) = \sum_i x_i \mathrm{P}(X = x_i)$	expectation of a discrete random variable X	1, 2, 3 •
$\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$	expectation of a continuous random variable X with p.d.f. f	2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \sum_i (x_i - \mu)^2 \mathrm{P}(X = x_i)$ $= \sum_i x_i^2\,\mathrm{P}(X = x_i) - \mu^2$	variance of a discrete random variable X	1, 2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \int (x - \mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$	variance of a continuous random variable X with p.d.f. f	2, 3 •
$\mathrm{F}(x) = \mathrm{P}(X \leqslant x) = \int_{-\infty}^{x} \mathrm{f}(x)\,\mathrm{d}x$	cumulative distribution function (c.d.f.)	2, 3 •

Random variables

Distribution	$\mathrm{P}(X = x)$	$\mathrm{E}(X)$	$\mathrm{Var}\,(X)$	**Papers**
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	1, 2, 3 •
Uniform distribution over $1, 2, \ldots, n$	$\frac{1}{n}$	$\frac{1}{2}(n+1)$	$\frac{1}{12}(n^2-1)$ (included for completeness; memorisation not required)	1, 2, 3 •
Poisson $\mathrm{Po}(\lambda)$	$\frac{\lambda^x \mathrm{e}^{-x}}{x!}$	λ	λ	2, 3 •

Distribution	p.d.f.	$\mathrm{E}(X)$	$\mathrm{Var}\,(X)$	**Papers**
Uniform distribution over $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$ (included for completeness; memorisation not required)	2, 3 •
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ (included for completeness; memorisation not required)	μ	σ^2	1, 2, 3 •